

Probability and statistics midterm exam (Model One)

A-Probability

Question (1) (10 points):

- A- A marble is drawn at random from a box containing 10 red, 30 white, 20 blue and 15 orange marbles. Find the probability that it is:
- (a) Orange or red, (b) not red or blue, (c) not blue, (d) white, (e) Red, white or blue.
- B- If A, B, C are independent events prove that (a) A and $B \cup C$, (b) A and $B \cap C$, (c) A and $B - C$, are independent.
- C- A box contains 3 blue and 2 red marbles while another box contains 2 blue and 5 red marbles. A marble drawn at random from one of the boxes turns out to be blue. What is the probability that it came from the first box?

Question (2) (10 points):

- A- Can the function $F(x)$ be a distribution function? Explain.
$$F(x) = \begin{cases} c(1 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$
- B- The joint probability function of two discrete random variables X and Y is given by $f(x, y) = cxy$ for $x = 1, 2, 3$ and $y = 1, 2, 3$, and equals zero otherwise. Find (a) the constant c, (b) $P(X = 2, Y = 3)$, (c) $P(1 \leq X \leq 2, Y \leq 2)$, (d) $P(X \geq 2)$, (e) $P(Y < 2)$, (f) $P(X = 1)$, (g) $P(Y = 3)$.

Question (3) (10 points):

- A- A bag contains one red and seven white marbles. A marble is drawn from the bag and its color is observed. Then the marble is put back into the bag and the contents are thoroughly mixed. Using (a) the binomial distribution and (b) the Poisson approximation to the binomial distribution, find the probability that in 8 such drawings a red ball is selected exactly 3 times.
- B- On a statistics examination the mean was 78 and the standard deviation was 10.
- (a) Determine the standard scores of two students whose grades were 93 and 62 respectively.
- (b) Determine the grades of two students whose standard scores were -0.6 and 1.2 respectively.

B- Statistics

Question (4) (10 points):

- Assume that the heights of 3000 male students at a university are normally distributed with mean 68.0 inches and standard deviation 3.0 inches. If 80 samples consisting of 25 students each are obtained.
- (a) What would be the expected mean and standard deviation of the resulting sampling distribution of means if sampling were done (i) with replacement, (ii) without replacement?
- (b) In how many samples would you expect to find the mean between 66.8 and 68.3 inches.

Question (5) (10 points):

- Suppose that the heights of 100 male students at XYZ University represent a random sample of the heights of all 1546 male students at the university. Determine unbiased and efficient estimates of (a) the true mean, (b) the true variance.